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Comments on "Some Effects of Planform Modification on the Skin Friction Drag"

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In a recent Technical Note, Hopkins[1] has presented an analysis of the skin friction drag for a "cranked" wing of double trapezoidal planform. The procedure followed includes consideration of the variation in skin friction coefficient with Reynolds number because of changes in wing chord.

It is interesting to note that this same basic concept was employed thirty years ago by Upson and the present writer[2] in a study of the drag of single trapezoidal planforms. The earlier work began with an expression for the profile drag coefficient of a chordwise element of the wing in the form

$$C_{D_0} = h[RN]^n(a_1 + a_2t^2)(1 + a_3C_L{}^3) \tag{1}$$

in which t is the thickness ratio and C_L the section lift coefficient. Definitions of the coefficients h, a_1, a_2, and a_3 are obvious from Eq. (1).

For the case corresponding to Hopkins' analysis, we put $t = 0$ and $C_L = 0$ and obtain

$$C_{D_0} = h[RN]^n a_1 \tag{2}$$

with the Reynolds number now given in terms of the local chord y as

$$RN = V_0 y/\nu \tag{3}$$

The application of Eq. (2) to the profile drag calculation for the chordwise strip of the wing $y\Delta x$ gives

$$C_{D_0} = [RN]_m{}^n \phi a_1 \tag{4}$$

where the Reynolds number is based on the mean chord and

$$\phi = \frac{2^{n+1}h(1 - K_y{}^{n+2})}{(n+2)(1+K_y)^{n+1}(1-K_y)} \tag{5}$$

Changing the notation so that $h = K$ and $K_y = 1/\lambda_2$, Eq. (5) becomes identical with that obtained by application of Hopkins' results in this restricted case.

In connection with Eq. (4) in the Hopkins paper, a slight error is to be noted in that the denominator of the term in the square brackets should read $(1 - \lambda_2)(\lambda_2)^{n+1}$ instead of $(1 - \lambda_2)(\lambda_2)^{n+2}$. This error apparently was not carried over into the computations leading to Figs. 2 and 3 of Ref. 1.

It may also be of interest to note that the earlier work of Ref. 2 included effects of section thickness ratio and lift coefficient as indicated by Eq. (1). Induced drag was also included in these considerations along with a combined treatment of both aerodynamic and structural factors.

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Comments on "Turbulent Mixing of Coaxial Jets"

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In the case of incompressible, fully developed, turbulent jet flows, Alpinieri[1] has stated that the conventional eddy viscosity assumption, $\epsilon = \kappa b|u_c - u_e|$ (where b is the jet half-width, u_e the constant external velocity, and u_c the centerline velocity), is only verified experimentally for the case $u_e = 0$, and therefore it is fair game to propose an entirely new assumption when $u_e > 0$. In an attempt to include the effects of variable density, he has apparently selected an assumption out of a set of possibilities all deemed equally probable but one that seems to fit his particular set of data.

However, in the far downstream region there is nothing to distinguish between wakes and jets (where $u_e > 0$, $u_c < u_e$) so that, for the constant density case, we find from the data of Cooper and Lutzky[2] and Carmody[3] that, for large x, $b \sim x^{1/3}$ and $u_e - u_c \sim x^{-2/3}$ which is in agreement with predictions based on the conventional eddy viscosity assumption[4] or on similarity considerations.[2] It follows from this that $\epsilon \sim x^{-1/3}$. Alpinieri's assumption, where in fact ϵ increases with x, is therefore in disagreement with this data.

Both Alpinieri and, in a related paper, Ferri, Libby, and Zakkay[5] object to the fact that, when $u_e = u_c$ and $\rho_e \neq \rho_c$, the conventional assumption leads to zero mixing of scalar quantities such as temperature or concentration. However, in the absence of upstream turbulence or velocity distortions this would, indeed, be evident to an observer travelling with the velocity $u = u_e = u_c$.

The answer seems to be that Refs. 2 and 3 cover a large range of x/d whereas Ref. 1 does not. Thus, Alpinieri is really dealing with the near downstream region with an assumption that, for the most part, involves variables appropriate to the far downstream region. The fact that mixing occurs when $u_e = u_c$ indicates that the turbulence is either present in the upstream flow or is subsequently generated by the velocity profile distortion due to the upstream boundary layers or both.

In the far downstream region where mixing is caused by turbulence generated by the jet or wake velocity profile, it seems evident that $\epsilon = \varkappa b|u_e - u_c|$ where, however, $\varkappa = \varkappa(\rho_c/\rho_e)$ and is still to be determined.†

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† A possible clue might be contained in the analysis of the inviscid stability of a discontinuous jet or cylindrical vortex sheet; one finds that a small disturbance grows like $\exp(\sigma t)$ where $\sigma = [(\rho_e\rho_c f)^{1/2}/(\rho_e + \rho_c f)]k|u_c - u_e|$, k is the disturbance wave number, $f = I_0(kb)K_1(kb)/I_1(kb)K_0(kb)$ (Bessel function notation as used by Hildebrand), and b = jet diameter. The implication is that one might expect $\epsilon \sim \sigma b^2$ where $kb = O(1)$ and $f = O(1)$.